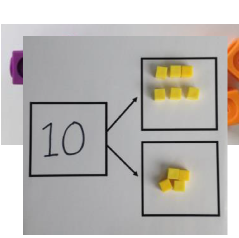
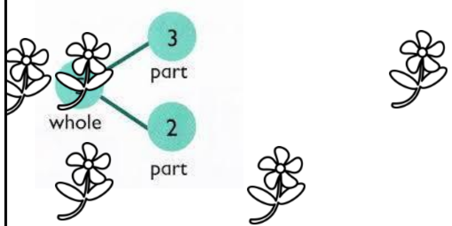
Addition

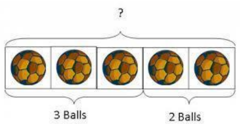
<table>
<tr><th>Objective and Strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Combining two parts to make a whole: part-whole model

R

(Y1)</td>
<td>

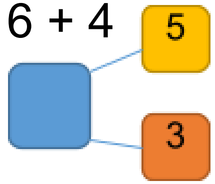
Use cubes to add two numbers together as a group or in a bar.

Numicon can also be used for this. (R)</td>
<td>



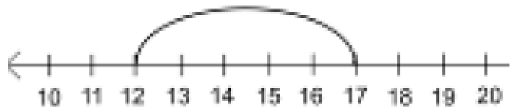
Use pictures to add two numbers together as a group or in a bar.

</td>
<td>4 + 3 = 7

10= 6 + 4


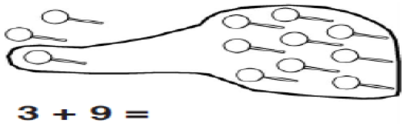
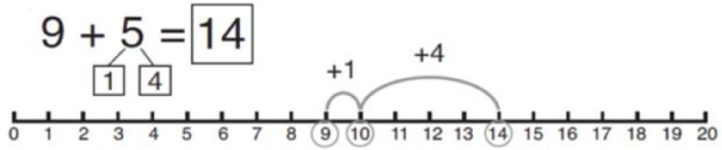
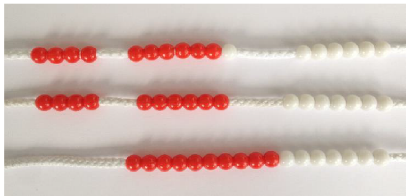
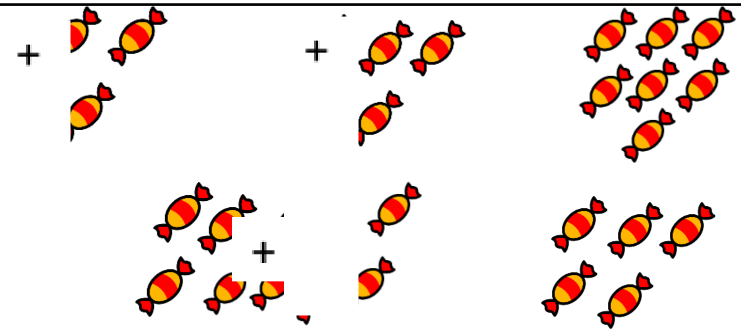
Use the part-part whole diagram as shown above to move into the abstract (mental).</td>
</tr>
<tr>
<td>Starting at the bigger number and counting on

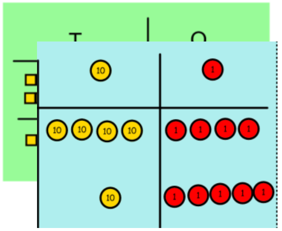
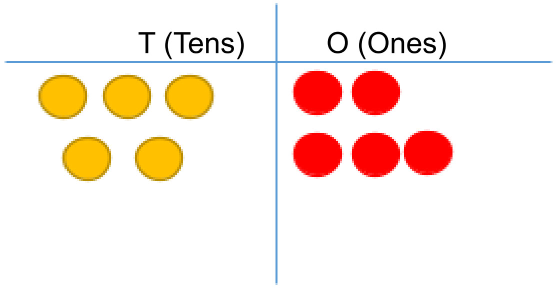
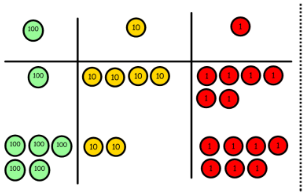
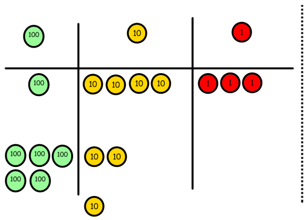
(R) Number lines
(Y1)</td>
<td>
Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

This could also be modelled with counters on a number track. Or with multilink towers.</td>
<td>12 + 5 = 17


Start at the larger number on the number line and count on in ones or in one jump to find the answer.</td>
<td>5 + 12 = 17

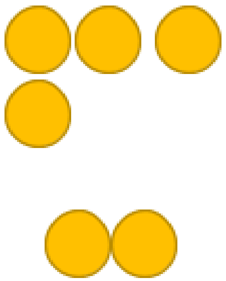
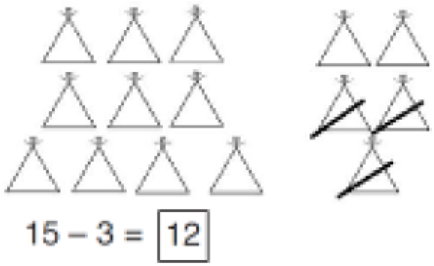
Record the larger number and count on the smaller number to find your answer.</td>
</tr>
</table>

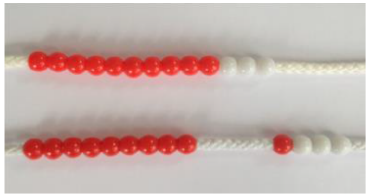
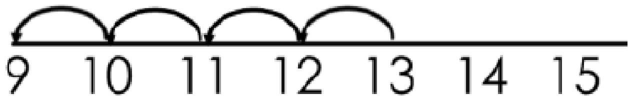
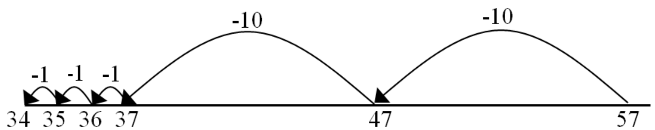
	(Number tracks used in number work and play in EYFS is preparation to support this learning)	(R) Reception will use number lines to identify numbers and will count on and back on them.	
Regrouping to make 10. (R) Use 5s and 10s frames (Y1)	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Tens frames are ideal. Counters and Numicon pieces also show this.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. Use a number track rather than a number line in early stages. This number line is one example of a pictorial representation.  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits (Y2)	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Numicon and ten frames also illustrate this effectively. Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to make the groups add to 10 first.	$4 + 7 + 6 = \boxed{10} + \boxed{7}$ 10 $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.

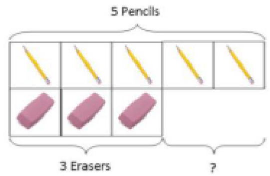
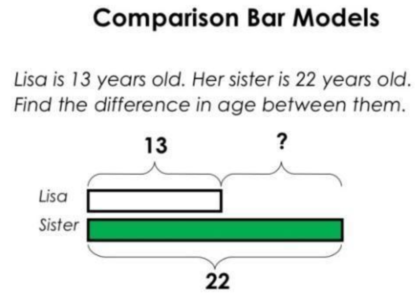
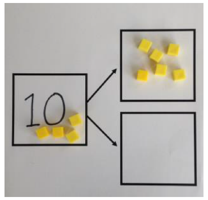
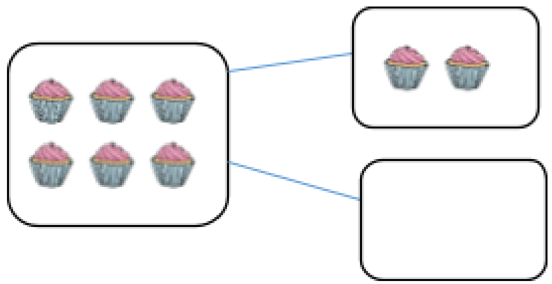
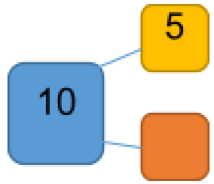
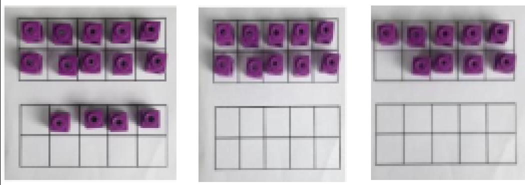
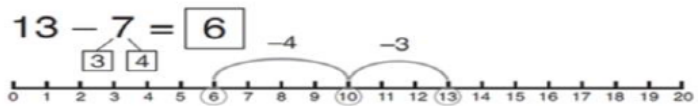
Column method- no regrouping (Y2)	Add together the ones first then add the tens. **Use the Base 10 blocks first before moving onto place value counters.** 24 + 15= 44 + 15 =  Ensure children understand the relative size of numbers before introducing place value counters.	After practically using the Base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42
Column method- regrouping Y3 – up to 3 digit numbers Y4 – up to 4 digit numbers Y5 – numbers with more than 4 digits	Make both numbers on a place value grid.  146 + 527 Add up the units and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. **This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.**	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. Pictorial representations can also be done with Dienes equipment.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. The expanded form supports reasoning and depth of understanding of the methods. They can be modelled side by side. 20 + 5 40 + 8 60 + 13 = 73 536 85 1 1 621 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. **Note that in**

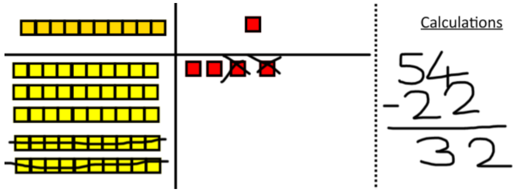
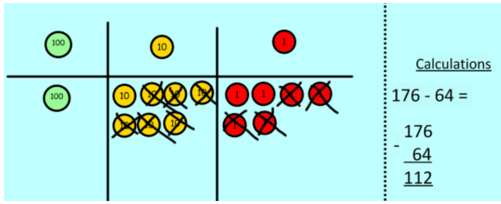
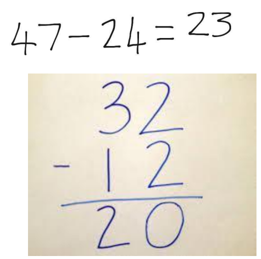
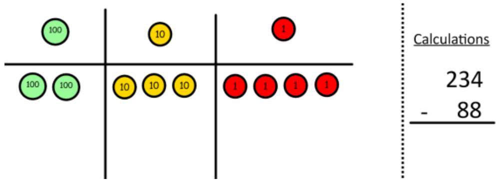
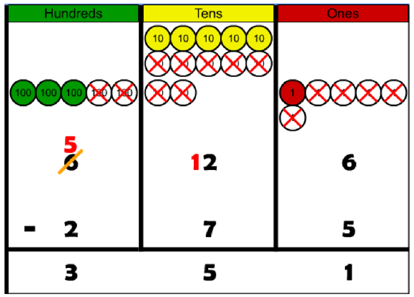
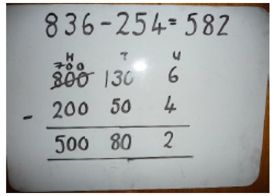
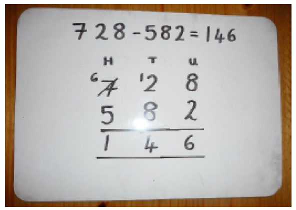
and decimals Y6 – all of the above and decimals with different decimal places	As children move on to decimals, money and decimals place value counters can be used to support learning.	**Use Base 10 until children have a sound grasp of the relative size between Th/H/T/O. Only then should children move onto place value counters.**	**regrouping we place the digits above the line ('on the doorstep').** 23.361 9. 080 59.770 2 2 2 93.111

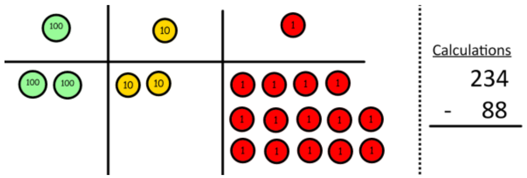
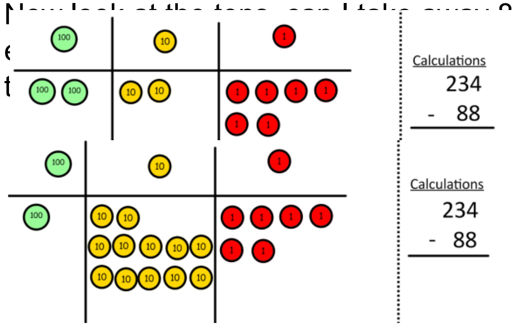
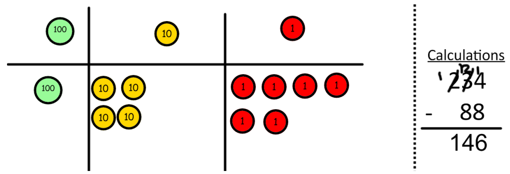
Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones (R) Concrete (Y1)	Use physical objects, counters, cubes etc. to show how objects can be taken away.  6 - 2 = 4	Cross out drawn objects to show what has been taken away.  	18 - 3= 15 8 – 2 = 6

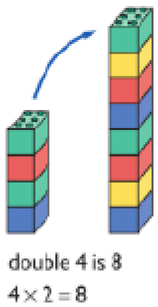
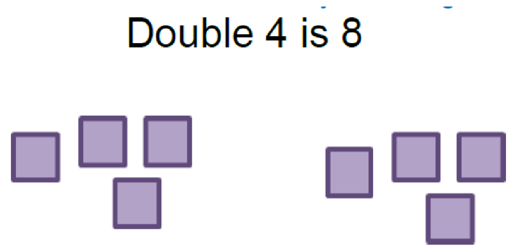
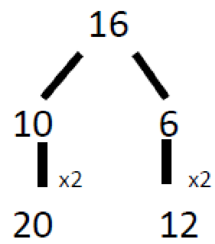
Counting back (Y1) (Y2)	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. Or lay them on a number track and remove them as you count back. 	Count back on a number line or number track.  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers. This can be revisited regularly in KS2 as part of fluency development.	Recall 13, count back 4. What number are you at? Use your fingers to help.

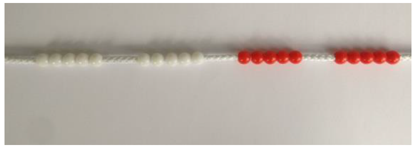
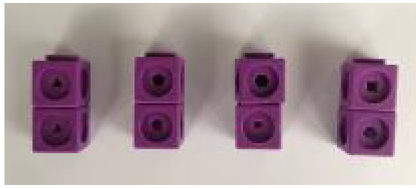
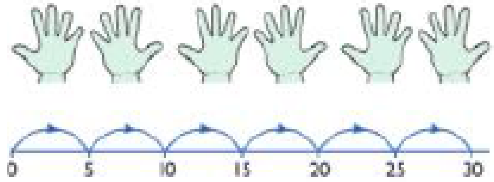
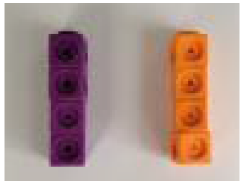
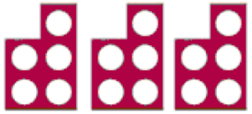
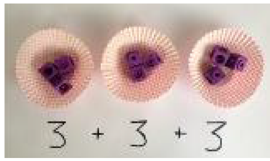
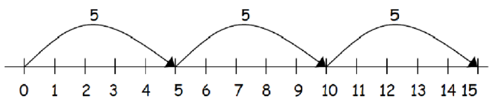
Find the difference (Y1) (Y2)	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar d the difference Numicon can also be used to find the difference by placing the pieces on top of each other.	Use 2 number lines to illustrate finding the difference e.g. 10 – 6: 1 10 0 6 Draw bars to find the difference between 2 numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the amounts of sandwiches.
Part, part whole model (Y1) (Y2)	 Linked to addition- use the part, part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 = Also use Cuisenaire to represent fact families in the bar model.	Use a pictorial representation of objects (or resources to represent the objects) to show the part, part whole model. 	 Move to using numbers within the part whole model.
Make 10 (Y1) (Y2)	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 then takeaway one	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. This can also be demonstrated with Numicon.	16 – 8 = How many do we take off to reach the next 10? How many do we have left to take off?

	more so you have taken away 5. You are left with the answer of 9.		
Column method without regrouping (Y2)	Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.	 Draw the Base 10 or place value counters alongside the written calculation to help to show working.  DIENES (BASE-10) SHOULD ALWAYS COME BEFORE PLACE VALUE COUNTERS.	 This will lead to a clear written column subtraction.
Column method with regrouping Y3 – up to 3 digit numbers Y4 – up to 4 digit numbers	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters.  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.	 Children can start their formal written method by partitioning the number into clear place value columns. (EXPANDED METHOD FIRST). 

Y5 – numbers with more than 4 digits and decimals Y6 – all of the above and decimals with different decimal places	 Now I can subtract my ones.  Now I can take away eight tens and complete my subtraction.  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.	 When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	Moving forward the children use a more compact method. (EXPANDED AND COMPACT CAN BE MODELLED SIDE BY SIDE). This will lead to an understanding of subtracting any number including decimals. $\begin{array}{rcccc} & 2 & {}^{5}\cancel{6} & {}^{12}\cancel{3} & . & {}^{1}0 \\ - & & 2 & 6 & . & 5 \\ \hline & 2 & 3 & 6 & . & 5 \end{array}$

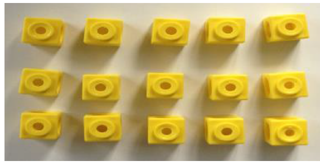
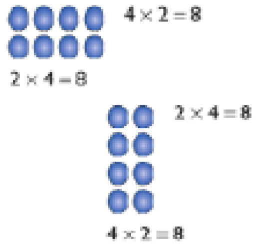
Multiplication

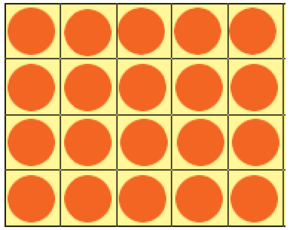
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling (R) Concrete and finger doubles to 10 (Y1) (Y2)	Use practical activities to show how to double a number.  This can also be done with Numicon reflections in mirrors, dominoes, dice…	Draw pictures to show how to double a number.  Can also be represented in a part, part whole model. Or in a bar model alongside a number line or Multilink cubes.	How could we work out 16 x 2 (same as doubling)  Partition a number and then double each part before putting it back together.

Counting in multiples (R) Counting in 2s, 5s and 10s (Y1) (Y2) (Y3)	  Count in multiples supported by concrete objects in equal groups. Numicon and Cuisenaire can support this.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Repeated addition (Y2) (Y3)	  Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

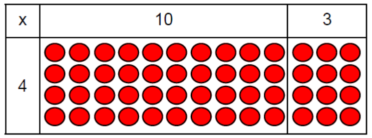
<table>
<tr>
<td>Arrays- showing commutative multiplication

(Y2)
(Y3)
(also continue to use arrays in Y4-6 to investigate factors, square numbers and prime numbers).</td>
<td>Create arrays using counters/ cubes to show multiplication sentences.

</td>
<td>Draw arrays in different rotations to find commutative multiplication sentences.



Link arrays to area of rectangles.</td>
<td>Use an array to write multiplication sentences and reinforce repeated addition.

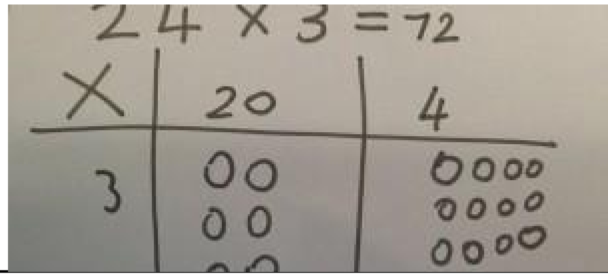
5 + 5 + 5 = 15
3 + 3 + 3 + 3 + 3 = 15
5 x 3 = 15
3 x 5 = 15</td>
</tr>
<tr>
<td>Grid Method

(Y3)</td>
<td>Show the link with arrays to first introduce the grid method.


4 rows of 10
4 rows of 3

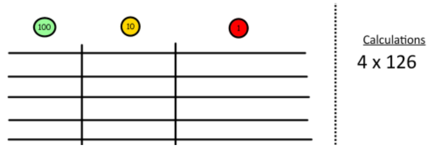
Move on to using Base 10 to move towards a more compact method.

4 rows of 13</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.

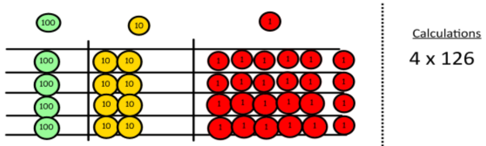
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

</td>
<td>Start with multiplying by one-digit numbers and showing the clear addition alongside the grid.
<table><tr><td>×</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245

Moving forward, multiply by a 2-digit number, showing</td>
</tr>
</table>

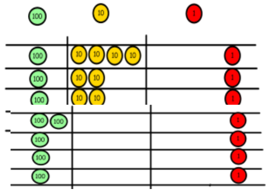
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children work with Dienes initially. They move to counters when they have a secure understanding of the relative size and place value.

the different rows within the grid method.

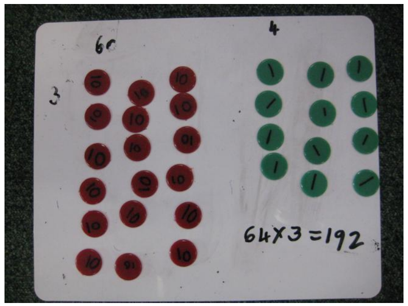
	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

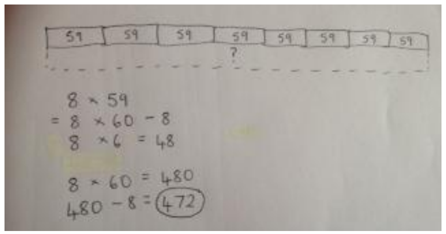
Column multiplication

Y4 – 2 and 3-digit numbers multiplied by a 1-digit number

Children can continue to be supported by place value counters at the stage of multiplication.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods. Children need to understand multiplication as repeated addition of equal groups in order to use the bar model for multiplication problem solving.



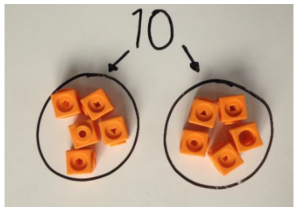
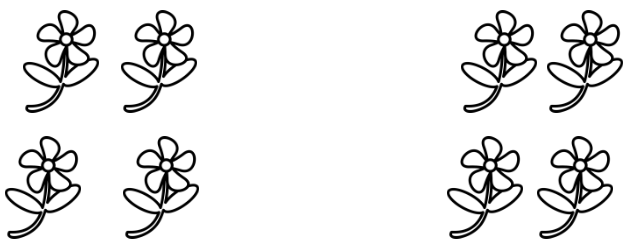
Short multiplication in Y4 and Y5, moving to long multiplication in Y5/6.

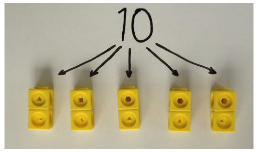
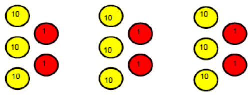
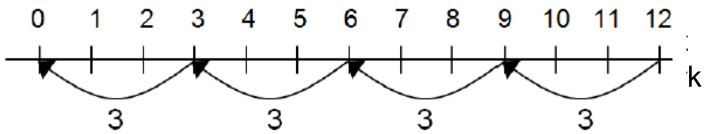
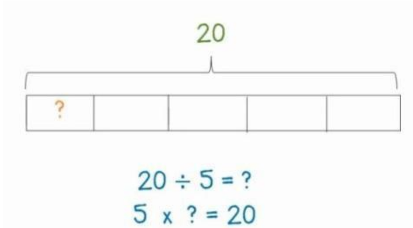
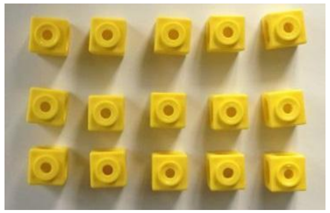
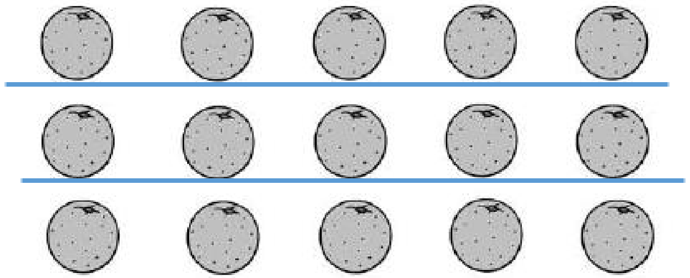
Start with expanded form, reminding the children about lining up their numbers clearly in columns.

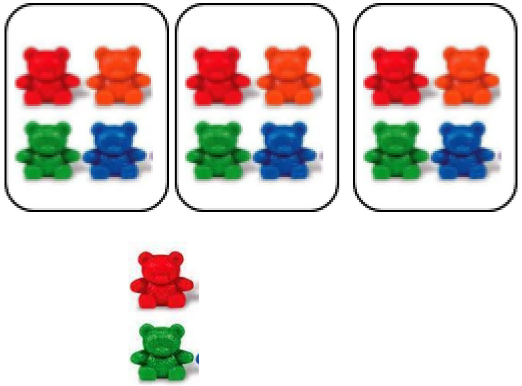
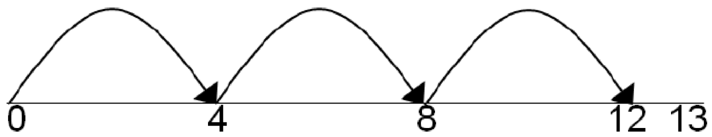
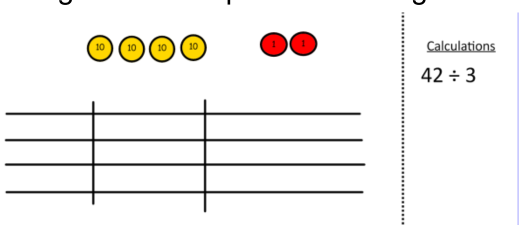
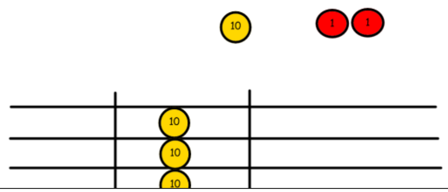
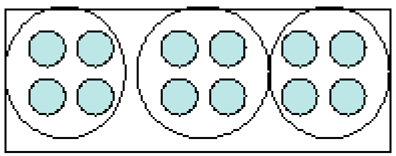
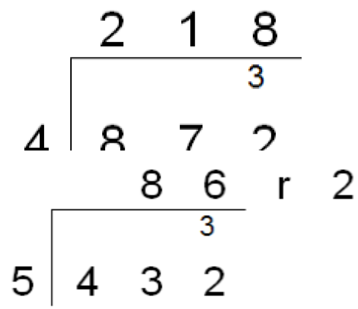
If it helps, children can write out what they are solving next to their answer.

Y5 – numbers with up to 4-digits multiplied by 1 or 2-digits Y6 – numbers with up to 4-digits multiplied by a 2-digit number	It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. Any child who is not accessing abstract column method can be supported with Dienes or place value counters as in the grid method model.	Bar models and number line representations of multiplication can also be applied to ratio and proportion in Y6.	120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 7 4 × 6 3 1 2 2 1 0 2 4 0 + 4 2 0 0 4 6 6 2 This moves to the more compact method. 2 3 1 1 3 4 2 x 1 8 1 3 4 2 0 1 0 7 3 6 2 4 1 5 6 1

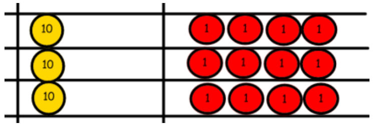
Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups (R) Mainly doubling and halving	 I have 10 cubes; can I share them equally into 2 groups?  **This picture illustrates 10 ÷ 2 as <u>sharing.</u>**	Children use pictures or shapes to share quantities.  $\boxed{8 \div 2 = 4}$	Share 9 buns between three people. 9 ÷ 3 = 3

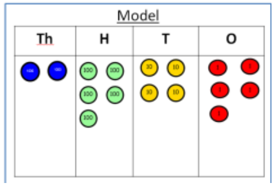
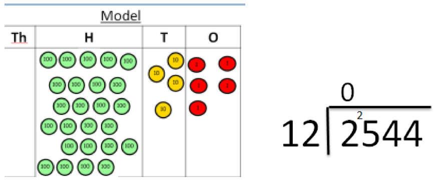
(Y1)			
Division as grouping (Y1) (Y2) Division by grouping is the division model which matches ÷ through times tables	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. **This picture illustrates 10 ÷ 2 as grouping.**   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays (Y2) (Y3) (Y4)	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences (fact families). 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

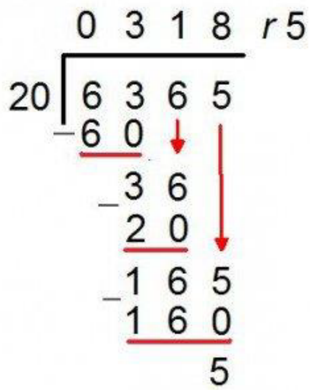
Division with a remainder (Y3) (Y4)	14 ÷ 3 = Divide objects between groups and see how much is left over  **THIS IS THE SHARING MODEL OF 14 ÷ 3.**	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  This model could also be represented using arrays. Draw dots and group them to divide an amount and clearly show a remainder.  **THIS IS THE GROUPING MODEL OF 14 ÷ 3.**	Complete written divisions and show the remainder using 'r'. 29 ÷ 8 = 3 REMAINDER 5 ↑ ↑ ↑ ↑ dividend divisor quotient remainder
Short division Y3 – 2-digits by 1-digit, taught through concrete and pictorial representations Y4 – up to 3-digit numbers divided by a 1-digit number, taught through concrete and pictorial representations	Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value; we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Use this only for small numbers. Encourage them to move towards counting in multiples to divide more efficiently. A pictorial representation would be children drawing the counters and the groups. As soon as they understand, move on to the abstract.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.

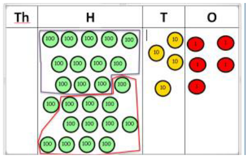
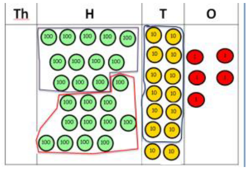
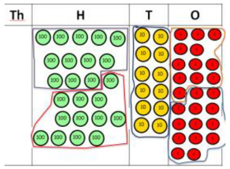
<table>
<tr>
<td>Y5 – up to 4-digit numbers divided by a 1-digit number, interpreting the remainder as appropriate, for the context of the problem

Y6 – As Y5 and also interpreting remainders as whole numbers/ fractions/round up or down</td>
<td>We exchange this ten for ten ones and then share the ones equally among the groups. 

We look how much is in 1 group and the answer is 14.</td>
<td></td>
<td>Finally move into decimal places to divide the total accurately.

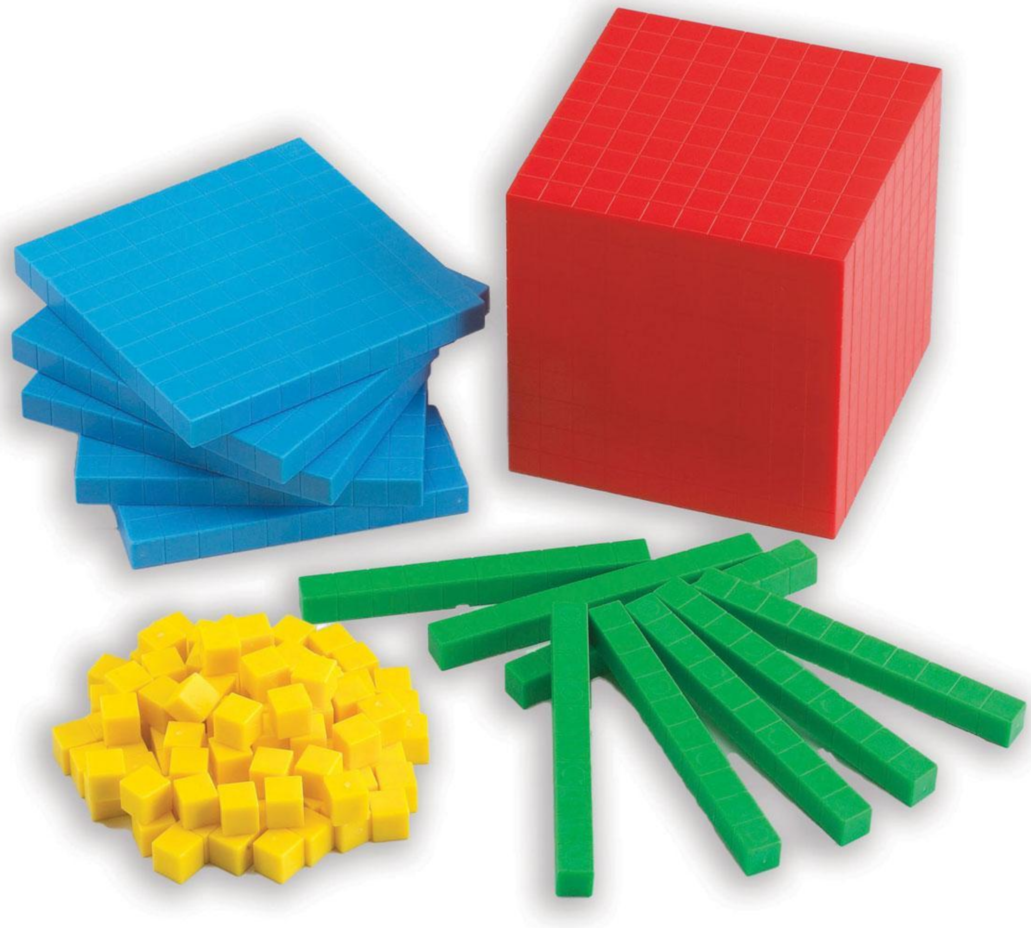
$$\begin{array}{r|l} & 1\ 4\ .\ 6 \\ \hline 3\ 5 & 5\ 1\ {}^{16}1\ .\ {}^{21}0 \end{array}$$</td>
</tr>
<tr>
<td>Long Division</td>
<td>
2544 ÷ 12
How many groups of 12 thousands do we have? None
Exchange 2 thousand for 20 hundreds.


How many groups of 12 are in 25 hundreds? 2 groups. Circle them.</td>
<td>Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.
Use this method to explain what is happening and as soon as they have understood this, move on to the abstract method as this can be a time consuming process.</td>
<td></td>
</tr>
</table>

	We have grouped 24 hundreds so can take them off and we are left with one.  0 2 12 ⟌ 2544 24 1 Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2  0 2 1 12 ⟌ 2544 24 14 12 2 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2  0 2 1 2 12 ⟌ 2544 24 14 12 24 24 0		

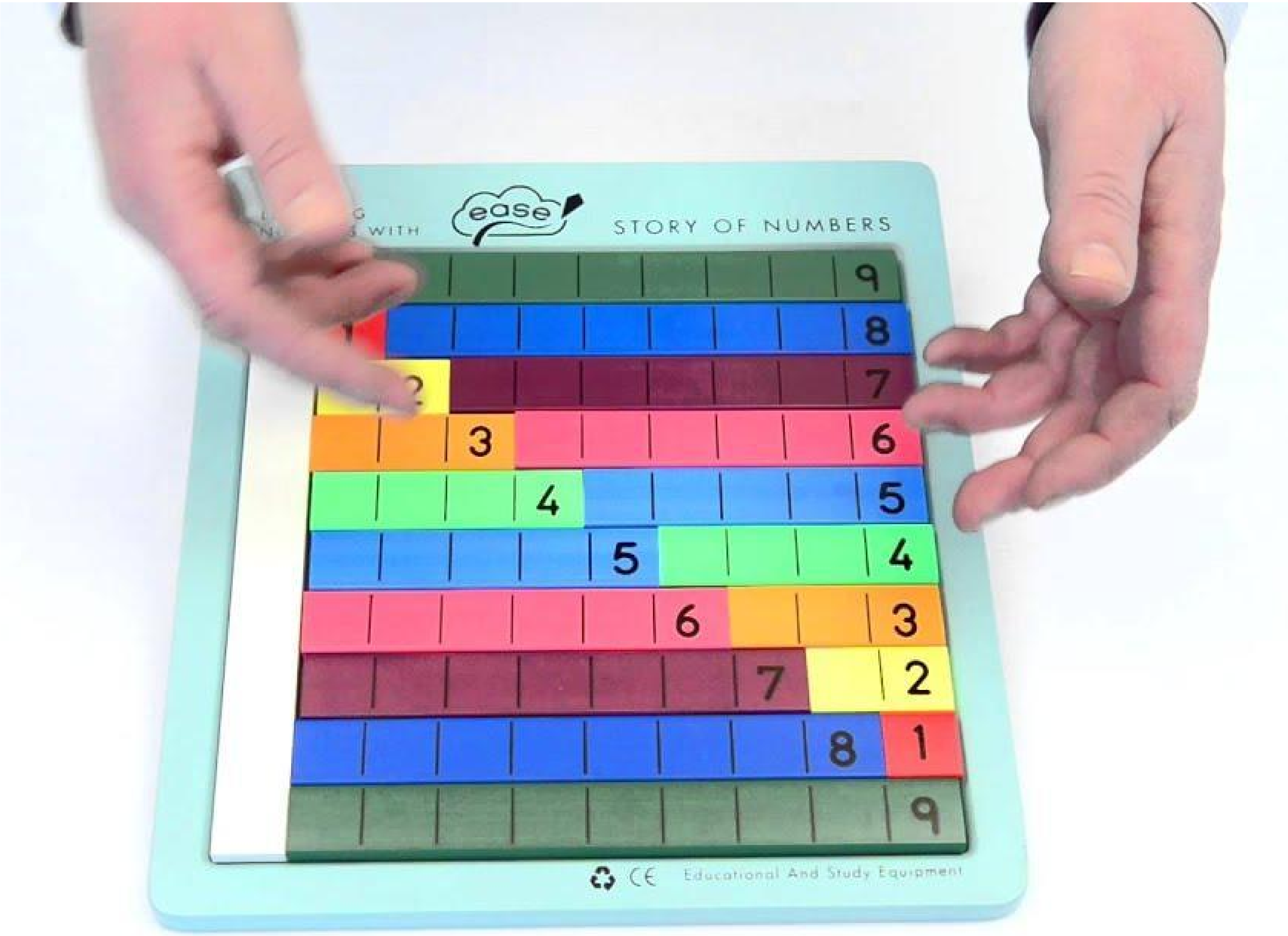
Resources that we use:

Dienes (Base-10)

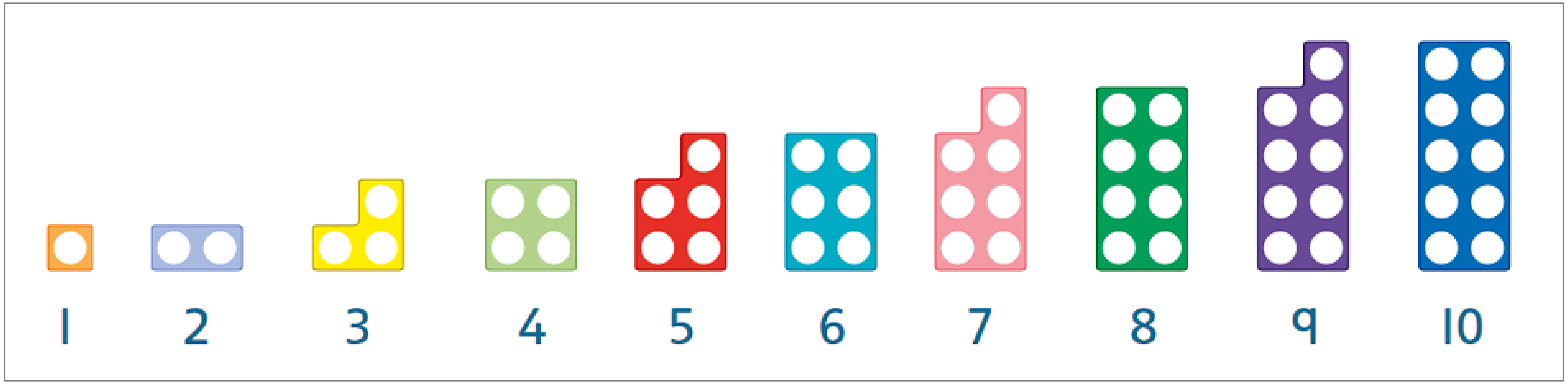


Cuisinaire

EASE apparatus



Numicon



Tens frames